A Proposal About the Rest Masses of Quarks

Jiao Lin Xu

The Center for Simulational Physics, The Department of Physics and Astronomy

University of Georgia, Athens, GA 30602, USA

E- mail: Jxu@Hal.Physast.uga.edu

Abstract

From the Dirac sea concept, we infer that a body center cubic quark lattice exists in the vacuum. Adapting the electron Dirac equation, we get a special quark Dirac equation. Using its low-energy approximation, we deduced the rest masses of the quarks: m(u)=930 Mev, m(d)=930 Mev, m(s)=1110 Mev, m(c)=2270 Mev and m(b)=5530 Mev. We predict new excited quarks $d_S(1390)$, $u_C(6490)$ and $d_b(9950)$.

I Introduction

The Standard Model [1] has been enormously successful in explaining and predicting a wide range of phenomena. In spite of the successes, the origin of quark masses is unknown [2]. In order to answer this fundamental question, since quarks are born from the vacuum, we will study the vacuum material. In a sense, the vacuum material works like a superconductor. Because the transition temperature is very high (much higher than the temperature at the center of the sun), there are no electric or mechanical resistances to any particle or to any physical body moving inside the vacuum material since they are moving under the transition temperature. They moving inside it look as if they are moving in completely empty space. The vacuum material is a super superconductor. From the Dirac sea concept [3], we infer (see Appendix) that there is

a body center cubic (BCC) quark lattice [4] in the vacuum. The quark lattice will help us to deduce the rest masses of the quarks.

The purpose of this proposal is to deduce the rest masses of the quarks. We do not discuss scattering and electroweak interactions. We will mainly discuss the low-energy strong interactions.

II Fundamental Hypotheses

Hypothesis I *: There are only two kinds of elementary quarks, u(0) and d(0), in the vacuum state. There are super-strong attractive interactions among the quarks (colors). These forces make and hold an infinite body center cubic (BCC) quark lattice with a periodic constant* $a \leq 10^{-18}m$ *in the vacuum.*

Hypothesis II *: Due to the effect of the vacuum quark lattice, fluctuations of energy* (ε) *and intrinsic quantum numbers (such as the Strange number* S*) of an excited quark will exist. The fluctuation of the Strange number is always* $\Delta S = \pm 1$ *[5].*

III The Special Quark Dirac Equation

According to the Fundamental **Hypothesis I**, there is a body center cubic quark lattice in the vacuum. When an excited quark (q) is moving in the vacuum, it is moving, in fact, inside the vacuum quark lattice. Physicists usually discuss Fermion problems based on the Dirac equation. The free particle Dirac equation [6] is

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(\overrightarrow{r},\mathrm{t}) =(\mathrm{i}\hbar \mathrm{c}\overrightarrow{\alpha}\cdot\overrightarrow{\nabla}\text{-}\beta \mathrm{mc}^2)\psi(\overrightarrow{r},\mathrm{t}), \tag{1}$$

where α are the α-matrices, β is the β-matrix and m is the rest mass of the free particle in the physical vacuum. Since this equation cannot discuss the effects of the vacuum quark

lattice, we must look for a new wave equation. We adapt the free electron Dirac equation (1) into a quark Dirac equation that can deal with the strong interactions between the excited quark(q) and the vacuum quark lattice based on the pure vacuum. First, we add two parts of Hamiltonian, H_{Latt} and H_{Accom} . H_{Latt} is the strong interactions (with body center cubic periodic symmetries) between the excited quark (q) and the vacuum quark lattice, and H_{Accom} is the strong interactions between the excited quark (q) and the two accompanying excited quarks (q'$_1$ and q'$_2$) [7]. Then we have to change the rest mass, m, of an electron in the physical vacuum into the bare mass [8], m_q, of an elementary quark in the pure vacuum. The quark Dirac equation will be

$$i\hbar\frac{\partial}{\partial t}\psi(\overrightarrow{r},t) = (H_0+H_{Latt}+ H_{Accom})\psi(\overrightarrow{r},t) \tag{2}$$

where H_0 ($i\hbar c\overrightarrow{\alpha}\cdot\overrightarrow{\nabla}$-$\beta m_q c^2$) is the free Hamiltonian of an elementary quark in the pure vacuum and m_q is the bare mass of the elementary quarks.

Since this is a multi-particle problem, it cannot be solved exactly. We take a mean-field approximation:

$$H_{Latt}+ H_{Accom} \simeq V_0, \tag{3}$$

where V_0 is a constant at any time-space point and in any reference frame; and

$$\boxed{\psi(\overrightarrow{r},t) \text{ satisfies BCC symmetries.}} \tag{4}$$

The above Dirac equation (2) will be approximated into

$$(i\hbar\frac{\partial}{\partial t}+ i\hbar c\overrightarrow{\alpha}\cdot\overrightarrow{\nabla}-\beta(m_q c^2+V_0)\psi(\overrightarrow{r},t) = 0. \tag{5}$$

Since $(i\hbar\frac{\partial}{\partial t}+i\hbar c\overrightarrow{\alpha}\cdot\overrightarrow{\nabla}-\beta m_q c^2)\psi(\overrightarrow{r},t) = 0$ is a free particle Dirac equation and V_0 is only a positive constant, the above quark equation (5) is Lorentz-invariant. We can prove, as with [9], that the equation (5) is Lorentz-invariant. We call it **the special quark Dirac equation**.

Using its low-energy approximation, we can deduce the rest masses of the quarks.

IV The Quarks

Our purpose is to deduce the rest masses of the quarks. Since the rest mass and the intrinsic quantum numbers of the quarks are the same in different reference frames, we will deduce them using the classic limit. It is sufficient to use the classic limit of the special quark Dirac equation– the quark Schrödinger equation [10],

$$\frac{\hbar^2}{2\mathrm{m}_q}\nabla^2\Psi + (\varepsilon\text{-}\mathrm{V}_0)\Psi = 0, \tag{6}$$

where m_q is the bare mass [8] of the elementary quark and V_0 is a constant. Using the quark Schrödinger equation we can deduce the rest masses of the quarks. These are needed by particle physics [11].

A The Unflavored Quarks (u and d)

If there were no BCC periodic symmetries (4), the solution of the above equation (6) is a free particle plane wave function of the quarks u or d:

$$\varepsilon_{\overrightarrow{k}} = \frac{\hbar^2}{2m_q}(\mathrm{k}_x^2+\mathrm{k}_y^2+\mathrm{k}_z^2)+\mathrm{V}_0, \quad \Psi_k = \exp(\mathrm{i}\,\overrightarrow{\mathrm{k}} \cdot \overrightarrow{r}). \tag{7}$$

If u is a free excited (from the vacuum) particle of the elementary u(0)-quark, it has the same intrinsic quantum numbers that the u(0)-quark has:

$$\mathrm{I} = \mathrm{I}_Z = \frac{1}{2}, \ \mathrm{S} = \mathrm{C} = \mathrm{b} = 0, \ \mathrm{Q} = \frac{2}{3}. \tag{8}$$

If d is a free excited (from the vacuum) particle of the elementary d(0)-quark, it has the same intrinsic quantum numbers that the d(0)-quark has:

$$\mathrm{I} = \text{-}\mathrm{I}_Z = \frac{1}{2}, \ \mathrm{S} = \mathrm{C} = \mathrm{b} = 0, \ \mathrm{Q} = \text{-}\frac{1}{3}. \tag{9}$$

When $\overrightarrow{k} = 0$, $\varepsilon_0 = \mathrm{V}_0 = \mathrm{m}_u = \mathrm{m}_d$. From the masses of the proton (uu'd') and the neutron (du'd') [7],

$$\mathrm{M}_p \sim \mathrm{M}_n \sim 940 \text{ Mev}, \tag{10}$$

and the accompanying excited quark masses ($m_{u'}$ = 3 Mev and $m_{d'}$= 7 Mev) [7] and [12], we get

$$m_u \approx m_d = V_0 = 930 \text{ Mev}. \qquad (11)$$

Now we have the rest masses of the unflavored quarks u and d. If we want to find the rest masses of the flavored quarks (s, c and b), we have to consider the BCC symmetries and the fluctuations of energies and strange numbers (**Hypothesis II**).

B The Flavored Quarks s, c and b

To deal with a particle moving in a lattice, physicists usually use energy band theory. According to the energy band theory [13], the solution of the quark Schrödinger equation, (6) satisfying the BCC symmetries, will be the Bloch [14] wave function

$$\psi_{\overrightarrow{k}}(\overrightarrow{r}) = e^{i\overrightarrow{k}\cdot\overrightarrow{r}}u(\overrightarrow{r}), \qquad (12)$$

where $\overrightarrow{k}$ is a vector in the space of the reciprocal lattice

$$\overrightarrow{k} = \overrightarrow{l}\mathbf{B}, \qquad (13)$$

$$\overrightarrow{l} = (l_1, l_2, l_3), \qquad (14)$$

l_1, l_2 and l_3 take all positive and negative integral values including zero,

$$\mathbf{B} = \mathbf{2}\pi \begin{pmatrix} 0 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{a} & 0 & \frac{1}{a} \\ \frac{1}{a} & \frac{1}{a} & 0 \end{pmatrix}, \qquad (15)$$

$\overrightarrow{k} = \overrightarrow{l}\,\mathbf{B}$ denotes any reciprocal lattice point; $u(\overrightarrow{r})$ is a periodic function [15]

$$u(\overrightarrow{r}+\mathbf{A}\overrightarrow{s}) = u(\overrightarrow{r}), \qquad (16)$$

$$\mathbf{A} = \begin{pmatrix} -\frac{a}{2} & \frac{a}{2} & \frac{a}{2} \\ \frac{a}{2} & -\frac{a}{2} & \frac{a}{2} \\ \frac{a}{2} & \frac{a}{2} & -\frac{a}{2} \end{pmatrix}. \tag{17}$$

We have

$$\mathrm{T}(\mathbf{A}\overrightarrow{S})\psi_{\overrightarrow{k}}\ (\overrightarrow{r}) = \mathrm{e}^{i\overrightarrow{k}\cdot\overrightarrow{r}}\mathrm{u}(\overrightarrow{r}).$$

Substituting the wave function (12) into the Schrödinger equation (6), we can get the equation of the periodic function $\mathrm{u}(\overrightarrow{r})$

$$\bigtriangledown^2\mathrm{u}(\overrightarrow{r})+2\mathrm{i}\,\overrightarrow{k}\cdot\bigtriangledown\mathrm{u}(\overrightarrow{r})+\frac{2\mathrm{m}_q}{\hbar^2}[(\varepsilon\ \text{-V}_0)-\frac{\hbar^2\mathrm{k}^2}{2\mathrm{m}_q}]\ \mathrm{u}(\overrightarrow{r})=0. \tag{18}$$

Since V_0 is a constant, a periodic solution of (18) is

$$\mathrm{u}(\overrightarrow{r}) = \mathrm{e}^{i\overrightarrow{l}\mathbf{B}\cdot\overrightarrow{r}} \tag{19}$$

for which the eigenvalue is

$$\varepsilon_l(\overrightarrow{k}) = \mathrm{V}_0 + \frac{\hbar^2}{2\mathrm{m}_q}\left|\overrightarrow{k} - \overrightarrow{l}\mathbf{B}\right|^2. \tag{20}$$

If taking

$$\vec{k} = (2\pi/\mathrm{a})(\xi,\ \eta,\ \zeta), \tag{21}$$

and substituting the B matrix (15) of the body center cubic quark lattice into (20), we get the energy

$$\varepsilon(\vec{k},\vec{n}) = \mathrm{V}_0+\alpha\mathrm{E}(\vec{k},\vec{n}), \tag{22}$$

$$\alpha = \mathrm{h}^2/2\mathrm{m}_q\mathrm{a}^2, \tag{23}$$

$$\mathrm{E}(\vec{k},\vec{n}) = (\mathrm{n}_1\text{-}\xi)^2+(\mathrm{n}_2\text{-}\eta)^2+(\mathrm{n}_3\text{-}\zeta)^2. \tag{24}$$

The solution of Eq. (6) is a plane wave

$$\psi_{\vec{k}}(\vec{r}) = e^{(-\ \mathrm{i}2\pi/\mathrm{a})[(\mathrm{n}_1\text{-}\xi)\mathrm{x}+(\mathrm{n}_2\text{-}\eta)\mathrm{y}+(\mathrm{n}_3\text{-}\zeta)\mathrm{z}]}, \tag{25}$$

where a is the periodic constant of the quark lattice and n_1, n_2 and n_3 are integers, $n_1 = l_2+l_3$, $n_2 = l_3+l_1$ and $n_3 = l_1+l_2$,

$$\begin{aligned} l_1 &= 1/2(\ \text{-}n_1+n_2+n_3), \\ l_2 &= 1/2(\ +n_1\text{-}n_2+n_3), \\ l_3 &= 1/2(\ +n_1+n_2\text{-}n_3), \end{aligned} \tag{26}$$

satisfying the condition that only those values of $\overrightarrow{n} = (n_1,\ n_2,\ n_3)$ are allowed, which make $\overrightarrow{l} = (l_1,\ l_2,\ l_3\ )$ an integer vector [16]. Condition (26) implies that the vector $\vec{n} = (n_1,\ n_2,\ n_3)$ can only take certain values. For example $\vec{n}$ cannot take (0, 0, 1) or (1, 1, -1), but it can take (0, 0, 2) and (1, -1, 2).

B- 1 The Energy Bands

Using the standard methods of energy band theory [13], we can deduce whole energy bands and wave functions for low energy bands, from (22) and (25). Since the purpose of this paper is deducing the rest masses of the quarks, we do not show the wave functions. Because quarks are all born on the single-energy bands of the Δ-axis and the Σ-axis, we

will discuss these single-energy bands only.

The single energy bands on the Δ-axis (Γ-H)

Energy Bands	n_1,n_2,n_3	R	d	J_Δ	E	Energy
E_Γ=0 → E_H=1	0, 0, 0	4	1	0	0	930
E_H=1 → E_Γ=4	0, 0, 2	4	1	J_H=1	1	1290
E_Γ=4 → E_H=9	0, 0, -2	4	1	J_Γ=1	4	2370
E_H=9 → E_Γ=16	0, 0, 4	4	1	J_H=2	9	4170
E_Γ=16 → E_H=25	0, 0, -4	4	1	J_Γ=2	16	6690
E_H=25 → E_Γ=36	0, 0, 6	4	1	J_H=3	25	9930
...	...	...	...	...	...	...

(27)

and

The single energy bands on the Σ-axis(Γ-N)

Energy Bands	n_1, n_2, n_3	R	d	J_Σ	E	Energy
E_Γ=0 → E_N=1/2	0, 0, 0	2	1	0	0	930
E_N=1/2 → E_Γ=2	1, 1, 0	2	1	J_N=1	$\frac{1}{2}$	1110
E_Γ=2 → E_N=9/2	-1, -1, 0	2	1	J_Γ=1	2	1650
E_N=9/2 → E_Γ=8	2, 2, 0	2	1	J_N=2	$\frac{9}{2}$	2550
E_Γ=8 → E_N=25/2	-2, -2, 0	2	1	J_Γ=2	8	3810
E_N=25/2 → E_Γ=18	3, 3, 0	2	1	J_N=3	$\frac{25}{2}$	5430
E_Γ=18 → E_N=49/2	-3, -3, 0	2	1	J_Γ=3	18	7410
E_N=49/2 → E_Γ=64	4, 4, 0	2	1	J_N=4	$\frac{49}{2}$	9750
...	...	...	...	...	...	...

(28)

B- 2 The Phenomenological Formulae of the Quantum Numbers

We have already found the energy bands that are shown in (27) and (28). These energy bands represent the excited states of the elementary quarks u(0) and d(0). These energy band excited states are the quarks and excited quarks. Comparing them with experimental data, we can recognize the quarks. For the first Brillouin zone, $\overrightarrow{n}$ = (0,

0, 0); it is a part of the free quark solution (7) and it has the lowest energy (mass). It represents the lowest mass u-quark (8) and the d-quark (9). Fitting the energy band excited states to the experimental results, we find the α in (22)

$$\alpha = \frac{\mathrm{h}^2}{2\mathrm{m}_q \mathrm{a}^2} = 360 \text{ Mev}. \tag{29}$$

Then we can find the formulae that we can use to deduce the quantum numbers and the rest masses of the energy bands, as shown in the following:

1. Baryon number B: When an elementary quark is excited (or accompanying excited [7]) from the vacuum state, it has

$$B = \frac{1}{3}. \tag{30}$$

2. Isospin number I: I is determined by the energy band degeneracy deg [13] with

$$\text{deg} = 2\text{I} + 1. \tag{31}$$

3. Strange number S: S is determined by the rotary fold R of the symmetry axis [13] with

$$S = R - 4, \tag{32}$$

where the number 4 is the highest possible rotary fold number of the BCC lattice.

4. Electric charge Q: The electric charge Q_q of the excited quark q is determined completely by the elementary quark (u(0) or d(0)) that is excited to produce the excited quark q. After getting B, I_Z and S (C and b), and considering the generalized Gell-Mann-Nishigima relationship [17], we can find the electric charges of the energy bands. If the $\mathrm{I}_z+\frac{1}{2}(\mathrm{B+S+C+b}) > 0$, it is an excited state of the elementary u(0)-quark,

$$\mathrm{I}_z+\frac{1}{2}(\mathrm{B+S+C+b}) > 0, \quad \mathrm{Q_q} = \mathrm{Q_u} = \frac{2}{3}; \tag{33}$$

otherwise, it is a excited state of the elementary d(0)-quark

$$\mathrm{I}_z+\frac{1}{2}(\mathrm{B+S+C+b}) > 0, \quad \mathrm{Q_q}= \mathrm{Q_d}= -\frac{1}{3}. \tag{34}$$

5. If a degeneracy (or subdegeneracy) of a group of energy bands is smaller than the rotary fold R,

$$\mathrm{deg} < \mathrm{R} \text{ and } \mathrm{R}\text{- deg} \neq 2, \tag{35}$$

then the formula (32) will be replaced by

$$\bar{\mathrm{S}} = \mathrm{R} - 4. \tag{36}$$

The real value of S is

$$\mathrm{S} = \bar{\mathrm{S}} + \Delta\mathrm{S} = \mathrm{S}_{Axis} \pm 1. \tag{37}$$

From Hypothesis II, $\Delta S = \pm 1$, we have a formula to deduce ΔS,

$$\Delta\mathrm{S} = [1\text{-}2\delta(\mathrm{S})]\mathrm{Sign}(\vec{n}), \tag{38}$$

where

$$\mathrm{Sign}(\vec{n}) = \frac{\mathrm{n_1+n_2+n_3}}{|\mathrm{n_1}|+|\mathrm{n_2}|+|\mathrm{n_3}|}. \tag{39}$$

6. The fluctuation of the strange number will be accompanied by an energy change (**Hypothesis II**). We assume that the change of the energy (perturbation energy) is proportional to (-ΔS) and a number, J, representing the energy level, as a phenomenological formula:

$$\Delta\varepsilon = (\mathrm{S}+1)100(\mathrm{J+S})(-\Delta\mathrm{S}).$$

For a single energy band, J will take 1, 2, 3 and so forth from the lowest energy band to higher ones for each of the two end points of the symmetry axes respectively.

7. Charmed number C and Bottom number b: The "Strange number," S, in (37) is not completely the same as the strange number in (32). In order to compare it with the experimental results, we would like to give it a new name under certain circumstances.

If S = +1, which originates from the fluctuation $\Delta S = +1$,

$$\text{then we call it the Charmed number } C \ (C = +1); \tag{40}$$

if S = -1, which originates from the fluctuation $\Delta S = +1$, and there is an energy fluctuation,

$$\text{then we call it the Bottom number } b \ (b = -1). \tag{41}$$

8. We assume that the excited quark's rest mass is the minimum energy of the energy band that represents the quark:

$$m_{q^*} = \text{Minimum}[\text{V}_0 + \alpha\text{E}(\overrightarrow{k}, \overrightarrow{n})] + \Delta\varepsilon. \tag{42}$$

This formula (42) is the united mass formula that can give the masses of all quarks.

B- 3 The Recognition of the Quarks

Using the above formulae (30) - (42), we can find the quantum numbers and masses of the energy bands. Using the quantum numbers and the masses, we can recognize the quarks (ground and excited quarks). Since the purpose of this paper is deducing the rest masses of the quarks, we do not show the whole quark spectrum. Because quarks are all born on the single-energy bands of the Δ-axis and the Σ-axis, we will discuss the quarks on the single-energy bands only.

1. The single-bands on the Δ-axis (Γ-H)

For the single-bands on the Δ-axis, R=4, S_Δ = 0 from (32); d =1, I =0 from (31). Since d = 1 < R = 4 and R-d = 3 $\neq$ 2, according to (35), we will use (37) instead of (42). Using (40), we have

Energy Band	n_1,n_2,n_3	S_Δ	ΔS	J	$\Delta\varepsilon$	S	C	q(m)
E_Γ=0 $\to$ E_H=1	0, 0, 0	0	0	0	0	0	0	u(930)
E_H=1 $\to$ E_Γ=4	0, 0, 2	0	-1	J_H=1	100	-1	0	d_S(1390)
E_Γ=4 $\to$ E_H=9	0, 0, -2	0	+1	J_Γ=1	-100	0	1	u_C(2270)
E_H=9 $\to$ E_Γ=16	0, 0, 4	0	-1	J_H=2	200	-1	0	d_S(4370)
E_Γ=16 $\to$ E_H=25	0, 0, -4	0	+1	J_Γ=2	-200	0	1	u_C(6490)
E_H=25 $\to$ E_Γ=16	0, 0, 6	0	-1	J_H=3	300	-1	0	d_S(10230)
...	...	...	...	...	...	...	...	...

(43)

2. The single bands on the axis Σ(Γ-N)

For the single bands on the Σ-axis, R=2, S_Σ = -2 from (32); d =1, I =0 from (31). Since d = 1 < R = 2 and R-d = 1 $\neq$ 2, according to (35), we should use (37) instead of (32). Using (41), we have

Energy Band	n_1, n_2, n_3	S_Σ	ΔS	S	J	q(m)
E_Γ=0 $\to$ E_N=$\frac{1}{2}$	0, 0, 0	0	0	0	0	d(930)
E_N=$\frac{1}{2}$ $\to$ E_Γ=2	1, 1, 0	-2	1	-1	J_N=1	d_S(1110)
E_Γ=2 $\to$ E_N=$\frac{9}{2}$	-1, -1, 0	-2	-1	-3	J_Γ=1	d_Ω(1650)
E_N=$\frac{9}{2}$ $\to$ E_Γ=8	2, 2, 0	-2	1	-1	J_N=2	d_S(2550)
E_Γ=8 $\to$ E_N=$\frac{25}{2}$	-2, -2, 0	-2	-1	-3	J_Γ=2	d_Ω(3810)
E_N=$\frac{25}{2}$ $\to$ E_Γ=18	3, 3, 0	-2	1	-1	J_N=3	d_b(5530)
E_Γ=18 $\to$ E_N=$\frac{49}{2}$	-3, -3, 0	-2	-1	-3	J_Γ=3	d_Ω(7310)
E_N=$\frac{49}{2}$ $\to$ E_Γ=32	4, 4, 0	-2	1	-1	J_N=4	d_b(9950)
...	...	...	...	...	...	.

(44)

B- 4 The Quark Spectrum

Continuing the above procedure, we can find the lower energy excited states of the elementary quarks. From (8), (9), (11), (43) and (44), there are only five ground states: u(930), d(930), s(1110), c(2270) and b(5530). They are the quarks; the others are the excited states. Since the purpose of this paper is to deduce the rest masses of the quarks (about excited quarks will show in our next paper "The Baryon Spectrum"), we list the quarks as shown in Table 1:

Table 1. The quark Spectrum

Elementary quarks, u(0) and d(0), in the vacuum

The two accompanying excited quarks
u', S=C=b=0, I=s=$\frac{1}{2}$, I_z=$\frac{1}{2}$, Q=$\frac{2}{3}$, $m_{u'}$=3
d', S=C=b=0, I=s=$\frac{1}{2}$, I_z=$\frac{-1}{2}$, Q=$\frac{-1}{3}$, $m_{d'}$=7

The Quarks (The Ground States)

quark	**u**	**d**	**s**	**c**	**b**
m	930	930	1110	2270	5530
S	0	0	-1	0	0
C	0	0	0	1	0
b	0	0	0	0	-1
I	$+\frac{1}{2}$	$+\frac{1}{2}$	0	0	0
I_Z	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
Q	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$

C The Baryons

Any excited q is always accompanied by two accompanying excited quarks (q'$_1$ and q'$_2$) [7]. The baryon number of the three-quark system (qq'$_1$q'$_2$), from (30), equals the sum of the three quarks,

$$B_{qq'_1q'_2} = B_q + B_{q'_1} + B_{q'_2} = 1. \tag{45}$$

(45) means that the three-quark systems are the baryons. We give the most important baryons in Table 2:

Table 2. The Ground States of the Baryons

Theory Baryons	Quantum. No	Experiment	ΔM
Baryon(M) [q(m)q'$_1$q'$_2$]	S, C, b, I, Q	Name(M)	R
N^+(940) [u_N(930) u(3)'d(7)']	0, 0, 0, $\frac{1}{2}$, 1	p(938)	0.2
N^0(940) [d_N(930)u(3)'d(7)']	0, 0, 0, $\frac{1}{2}$, 0	n(940)	0.0
Λ_s^0(1120) [d_Λ(1110)u(3)'d(7)']	-1, 0, 0, 0, 0	Λ^0(1116)	0.4
Λ_c^+(2280) [u_C(2270)u(3)'d(7)']	0, 1, 0, 0, 1	Λ_c^+(2285)	0.2
Λ_b^0(5540) [d_b(5530)u(3)'d(7)']	0, 0, -1, 0, 0	Λ_b^0(5641)	1.8

In the fourth column, R =($\frac{\Delta M}{M}$)%.

The theoretical quantum numbers (I, S, C, B, Q) of the important baryons are completely the same with the experimental results. The theoretical masses of the important baryons agree well with the experimental results. These show that the theoretical masses of the quarks are correct.

D The Mesons

According to the Quark Model, a meson is made of a quark q_i and an antiquark $\overline{q_j}$. Since we have found the quark spectrum (see Table 1), using the sum laws, we can find the quantum numbers (S, C, b, I and Q) of the quark pairs (q_iq_j). Since there is not a theoretical formula for the binding energies, we propose a phenomenological formula for the binding energy. Because all quarks are the excited states of the elementary quarks u(0) and d(0), the binding energies are roughly constant (- 1671 Mev). If the differences between the quark mass and the antiquark mass in the quark pairs is larger, the binding energy is smaller ($\frac{\Delta m}{930}$):

$$E_B(q_i\overline{q_j}) = -1671+100[\frac{\Delta m}{930}+\Delta G+(C_i+\overline{C_j})]+50\delta(G_i)\delta(G_j), \tag{46}$$

where $\Delta m = |m_i - m_j|$; $\Delta G = |G_i+G_j|$ $\left|G_i+\overline{G_j}\right|$, G = S + C +b, S– strange number, C– charmed number and b– bottom number; $\delta(G_i)$ is Dirac function if $G_i = 0$ $\delta(G_i) = 1$ and if $G_i \neq 0$ $\delta(G_i) = 0$. Using (46) we can deduce the masses of the most important mesons as shown in Table 3:

Table 3. The Most Important Mesons

Experiment	$\overline{q_j(m_j)}q_i(m_i)$	E_{bind}	Theory	R
$\pi(139)$	$\overline{q_N(930)}q_N(930)$	-1721	$\pi(139)$	0
K(494)	$\overline{q_S(1110)}q_N(930)$	-1552	K(488)	1.2
$\eta(547)$	$\overline{q_S(1110)}q_S(1110)$	-1671	$\eta(549)$	0.4
D(1869)	$\overline{q_N(930)}q_C(2270)$	-1327	D(1878)	0.2
$D_S(1969)$	$\overline{q_S(1110)}q_C(2270)$	-1446	$D_S(1934)$	1.8
$J/\psi(3097)$	$\overline{q_C(2270)}q_C(2270)$	-1471	$J/\psi(3069)$	0.9
B(5279)	$\overline{q_b(5530)}q_N(930)$	-1076	B(5384)	2.0
$B_S(5344)$	$\overline{q_b(5530)}q_S(1110)$	-1196	$B_S(5444)$	1.9
$B_C(6400)$	$\overline{q_b(5530)}q_C(2270)$	-1220	$B_C(6580)$	2.8
$\Upsilon(9460)$	$\overline{q_b(5530)}q_b(5530)$	-1671	$\Upsilon(9389)$	0.8

In the fifth column, R $=(\frac{\Delta M}{M})\%$.

The theoretical quantum numbers (I, S, C, B, Q) of the important mesons are completely the same with the experimental results. The theoretical masses of the important mesons agree well with the experimental results. These show that the theoretical masses of the quarks are correct.

V Predictions

This proposal predicts some quarks and baryons.

A The Quarks

The new excited quarks: $d_S(1390)$, $u_c(6490)$ and $d_b(9950)$.

B The Baryons

For these new quarks, there will be the new baryons:

$[d_S^0(1390)u'(3)d'(7)] = \Lambda(1400)$ $\quad \Lambda(1405)$

$[u_C^0(6490)u'(3)d'(7)] = \Lambda_c^+(6500)$ $\quad ?$

$[d_b^0(9950)u'(3)d'(7)] = \Lambda_b^0(9960)$ $\quad ?$

The baryon $\Lambda(1405)$ has already been discovered by experiments. The baryons $\Lambda_c^+(6500)$ and $\Lambda_b^0(9960)$ are waiting to be discovered.

VI Discussions

1. From (23) and (29), we have

$$m_q a^2 = h^2/720 \text{ Mev}. \tag{47}$$

Although we do not know the values of m_b and a, we find that $m_b a^2$ is a constant. Since the Standard Model works fine without considering the quark lattice, we estimate a $\leqq 10^{-18}$m (the distance scales limit of the standard model) [1]. Thus the bare mass (m_q) of the elementary quarks is

$$\begin{aligned} m_q &= \frac{h^2}{720a^2} \geqq \frac{43.90\times 10^{-66}(\text{J s})^2}{720\times 10^6\times 1.602\times 10^{-19}\text{J} \times 10^{-36}\text{m}^2} \\ m_q &\geqq 3.8 \times 10^{-15}\text{kg} = 2.27 \times 10^{12} m_p \\ &\geqq 2.129 \times 10^{15}\text{Mev}. \end{aligned} \tag{48}$$

It is much larger than the excited quark rest masses. This ensures that the Schrödinger equation is a good approximation of the special quark Dirac equation for deducing the rest quark masses and that the Standard Model is an excellent approximation of the unborn fundamental theory at the distance scales $> 10^{-18}$m.

2. After the discovery of superconductors, we could understand the vacuum material. In a sense, the vacuum material (skeleton– the BCC quark lattice) works like a superconductor. Since the transition temperature is very high (much higher than the temperature at the center of the sun), all phenomena that we can see are under the transition temperature. Thus there are no electric or mechanical resistances to any particle or to any physical body moving inside the vacuum material. As they move (with a

constant velocity) inside it, they look as if they are moving in completely empty space. The vacuum material is a super superconductor.

3. There is an approximation (the quark Schrödinger equation instead of the low energy special quark Dirac equation) used in deducing the rest masses of the quarks. The theoretical baryon spectrum (Table 2) and the theoretical meson spectrum (Table 3) agree well with the experimental results. These strong agreements mean that this approximation is good. Since the rest masses are the same in any reference, in order to deduce the rest masses, we can use the Schrödinger equation instead of the special quark Dirac equation [10] to deduce the approximate rest masses.

4. The flavored quarks (the flavored baryons and mesons) originated from the body center cubic periodic symmetries of the BCC quark lattice and the fluctuation. If BCC symmetries did not exist, there would not be any flavored quark (baryon or meson).

5. The proposal has shown that the u-quark and the c-quark are the excited states of the elementary u(0)-quark and that the d-quark, the s-quark and the b-quark are the excited states of the elementary d(0)-quark. The u(0)-quark and the d(0)-quark have a SU(2) symmetry (u(0) and d(0)). Therefore SU(3) (u, d and s), SU(4) (u, d, s and c) and SU(5) (u, d, s, c and b) are correct although there are large differences in masses between the quarks. In fact, the SU(3), SU(4) and SU(5) are natural expansions of the SU(2). Since the bare masses of the elementary quarks u(0) and d(0) (48) are huge

$$\mathrm{m}_{\mathrm{u(0)}}(\text{or } \mathrm{m}_{\mathrm{d(0)}}) \ \geqq\ 2.27 \times 10^{12}\mathrm{m}_p = 2.129 \times 10^{15}\mathrm{Mev}, \tag{49}$$

thus the bare masses (taking the absolutely empty spece as the zero energy point) of the quarks (u, d, s, c, b, u' and d') are huge too:

$\mathrm{m}_u^{bare} = \mathrm{m}_{u(0)}+930$	$\mathrm{m}_d^{bare} = \mathrm{m}_{d(0)}+930$
$\mathrm{m}_c^{bare} = \mathrm{m}_{u(0)}+2270$	$\mathrm{m}_s^{bare} = \mathrm{m}_{d(0)}+1110$
$\mathrm{m}_b^{bare} = \mathrm{m}_{d(0)}+5530$	$\mathrm{m}_{u'}^{bare} = \mathrm{m}_{u(0)}+3$
$\mathrm{m}_d^{bare} = \mathrm{m}_{d(0)}+7$	.

(50)

From (48) and (50), we can see that the bare masses of the quarks (u, d, s, c, b, u' and d') are essentially the same. This is a rigorous physical basis of the SU(3), SU(4), SU(5) and so forth symmetries. We had been thinking that SU(4) and SU(5) do not have rigorous physical basis for a long time since the differences of the quark masses are too large. Considering the bare masses of the quarks, now we believe that SU(4) and SU(5) symmetries really exist.

6. After we study the BCC quark lattice (with a periodic constant a $\leq 10^{-18}$m) and deduce the masses and the intrinsic quantum numbers (I, s, c, b and Q) of the quarks, we have a deeper understanding of the Standard Model. When the distance scales $>$ $10^{-18}m$, although we cannot see the quark lattice, we will see the u(0)-quark Dirac sea and the d(0)-quark Dirac sea. Sometimes, we can also see the s-quark, the c-quarkand the b-quark (inside baryons and mesons); and, from the Dirac sea concept, we guess that there will be an s-quark Dirac sea, a c-quark Dirac sea and a d-quark Dirac sea too. Since we cannot see the quark lattice (we can only see the Dirac seas), we cannot deduce the masses and the intrinsic quantum numbers (experimental measurements). Naturally we think that the quarks (u, d, s, c and b) are all independent elementary particles. The Standard Model is a reasonably excellent approximation to nature at distance scales as small as 10^{-18}m [1]. Thus there is no contradiction between the Standard Model and the BCC quark lattice Model, otherwise the BCC quark lattice model does provide a physical basis for the Standard Model.

VII Conclusions

1. The origin of the quark masses is the strong interactions between the BCC quark lattice and the excited quarks. Using the strong interactions, we have deduced that m(u)=930 Mev, m(d)=930 Mev, m(s)=1110 Mev, m(c)=2270 Mev and m(b)=5530 Mev. The theoretical masses (deduced from the quark masses) of the most important baryons

(Table 2) and mesons (Table 3) agree well with the experimental results. This shows that the theoretical masses of the quarks are correct.

2. There are only two elementary quarks (u(0) and d(0)) in the vacuum state. All quarks (u, d, s, c, b ...) with rest mass $\neq 0$ are the energy band excited states of u(0) and d(0). The quarks u, d, s, c and b are the ground states of them. The quarks u(930) and $u_C(2270)$ are the excited states of the elementary u(0)-quark: the quarks d(930), s(1110) and b(5530) are the excited states of the elementary d(0)-quark.

3. The SU(3), SU(4) and SU(5) are natural expansions of the SU(2).

4. The BCC quark lattice model provides a rigorous physical basis for the Quark Model.

5. Due to the existence of the vacuum quark lattice, all observable particles are constantly affected by the lattice. Thus some laws of statistics (such as fluctuation) cannot be ignored.

Acknowledgment

I would like to express my heartfelt gratitude to Dr. Xin Yu for checking the calculations of the energy bands. I sincerely thank Professor Robert L. Anderson for his valuable advice. I acknowledge my indebtedness to Professor D. P. Landau for his help also.

References

[1] M. K. Gaillard, P. D. Grannis, and F. J Sciulli, Rev. Mod. Phys., 71 No. 2 Centenary (1999) S96.

[2] Fayyazuddin and Riazuddin, *A Modern Introduction To PARTICLE PHYSICS* (World Scientific, Singapore) (2000) xi.

[3] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford at the Clarendon Press, Fourth Edition, 1981) 273; L. H. Ryder, *QUANTUM FIELD THEORY* (CAMBRIDGE UNIVERSITY PRESS, New York, 1985) 45-48.

[4] R. A. Levy, *Principles of Solid State Physics* (Academic, New York, 1972) 41.

[5] K. Binder, *Monte Carlo Methods in Statistical Physics* (Springer-Verlag, Berlin 1979); D. P. Landau, Phys. Rev. B 14 (1979) 4054.

[6] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford at the Clarendon Press, Fourth Edition, 1981) 254.

[7] J. L. Xu, hep-ph/0401018.

[8] L. H. Ryder, *QUANTUM FIELD THEORY* (CAMBRIDGE UNIVERSITY PRESS, New York, 1985) 334.

[9] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964) Section 2.2.

[10] H. Grosse, A. Martin, *Particle Physics and the Schrödinger Equation (*University Press, CAMBRIDGE, 1997) 1-22.

[11] The Oecd Frum, *Particle Physics (Head of Publications Service, OECD) (1995) 55.* A. Pais, Rev. Mod. Phys., **71 No.2,** (1999) S16.

[12] Particle Data Group, Phys. Rew. **D66**, (2002) 01001-29.

[13] J. Callaway, Energy Band Theory, (Academic Press, New York and London, 1964) 1-11; H. Jones, *The Theory of Brillouin Zones and Electronic States in Crystals,* (NORTH-HOLLAND PUBLISHING COMPANY, AMSTERDAM, 1975) 117.

[14] F. Bloch, Z. Physik **52**, (1928) 5555; H. Jones, *The Theory of Brillouin Zones and Electronic States in Crystals*, (NORTH-HOLLAND PUBLISHING COMPANY, AMSTERDAM, 1975) 34.

[15] H. Jones, *The Theory of Brillouin Zones and Electronic States in Crystals*, (NORTH-HOLLAND PUBLISHING COMPANY, AMSTERDAM, 1975) 36.

[16] H. Jones, *The Theory of Brillouin Zones and Electronic States in Crystals*, (NORTH-HOLLAND PUBLISHING COMPANY, AMSTERDAM, 1975) 125.

[17] M. Gell-Mann, Phys. Rew. **92** (1953) 833; K. Nishijima, Prog. Theor. Phys. **12** (1954) 107.

[18] R. C. Evans, *An Introduction to Crystal Chemistry* (Cambridge, 1964) 41.

VIII Appendix: The Body Center Cubic Quark Lattice

According to Dirac's sea concept [3], there is an electron-Dirac sea, a μ-lepton Dirac sea, a τ-lepton Dirac sea, a u-quark Dirac sea, a d-quark Dirac sea, an s-quark Dirac sea, a c-quark Dirac sea, a b-quark Dirac sea and so forth in the vacuum. All of these Dirac seas are in the same space, at any location. These particles will interact with one another and form the perfect physical vacuum material. Some kinds of particles, however, do not play an important role in forming the physical vacuum material. First, the main force which makes and holds the structure of the physical vacuum material must be the strong interactions, not the weak-electromagnetic interactions. Hence, in considering the structure of the vacuum material, we leave out the Dirac seas of those particles which do not have strong interactions (e, μ and τ). Secondly, the physical vacuum material is super stable, hence we also omit the Dirac seas which can only make unstable baryons

(the s-quark, the c-quark and the b-quark). Finally, there are only two kinds of possible particles left: the vacuum state $u(0)$-quarks and the vacuum state $d(0)$-quarks. There are super strong attractive forces between the $u(0)$-quarks and the $d(0)$-quarks (colors) that will make and hold the densest structure of the vacuum material.

According to solid state physics [18], if two kinds of particles (with radius $R_1 < R_2$) satisfy the condition $1 > R_1/R_2 > 0.73$, the densest structure is the body center cubic lattice [4]. We know the following: first, u quarks and d quarks are not exactly the same, thus $R_u \neq R_d$; second, they are very close to each other (with the same bare masses essentially), thus $R_u \approx R_d$. Hence, if $R_u < R_d$ (or $R_d < R_u$), we have $1 > R_u/R_d > 0.73$ (or $1 > R_d/R_u > 0.73$). Therefore, we conjecture that the vacuum state u(0)-quarks and d(0)-quarks construct the body center cubic quark lattice in the vacuum.